MIXED-REGISTER STABILIZER CODES: A CODING THEORETIC PERSPECTIVE

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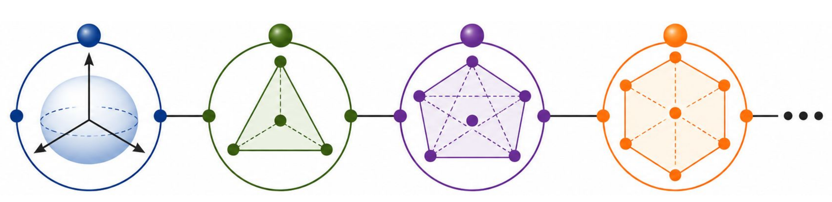
OBJECTIVES

We develop a coding-theoretic framework for stabilizer codes on *mixed-register* quantum devices whose quantum locations have non-uniform local-dimensions.



- We characterize the algebraic structure governing mixed-register Pauli commutation.
- We prove several no-go theorems.
- We construct good mixed-register qLDPC stabilizer codes using already known good qLDPC stabilizer codes.

DEFINITIONS

$\mathrm{Regdim}(D) = (Q_1, \ldots, Q_n)$ denotes the local-dimensions of an n register quantum device D. The symplectic representation $\phi : \mathcal{P}^n_{\mathrm{Regdim}(D)} \rightarrow \mathbb{Z}^{2n}_{\mathrm{Regdim}(D)}$ maps a Pauli operator to its X- and Z-exponents modulo the corresponding local-dimensions, written $\phi = (\phi_x \,|\, \phi_z)$. For $s_i, s_j \in \mathcal{P}^n_{\mathrm{Regdim}(D)}$, define the generalized symplectic inner product $\phi(s_i)\tilde{\odot}\phi(s_j)$

$$= \sum_{h=1}^{n} \frac{\phi_{x[h]}(s_i)\phi_{z[h]}(s_j) - \phi_{z[h]}(s_i)\phi_{x[h]}(s_j)}{Q_h}. \quad (1)$$

$S \subseteq \mathcal{P}^n_{\mathrm{Regdim}(D)}$ is a *mixed-register stabilizer code* iff $\phi(s_i)\tilde{\odot}\phi(s_j) \equiv 0 \pmod 1$ for all $s_i, s_j \in S$.

NO-GO THEOREMS

1. **Coprime trans-dimensional entangling gates are non-Clifford:** Clifford gates cannot entangle coprime local-dimensions.
2. **No true oscillator-qudit codes:** Mixed-register stabilizer codes over finite-dimensional and infinite-dimensional registers are equivalent to a tensor product of stabilizer codes over the finite-dimensional registers and the infinite-dimensional registers.
3. **Coprime codes must be disjoint:** Mixed-register stabilizer codes over pairwise coprime local-dimensions are equivalent to tensor products of stabilizer codes over the constituent local-dimensions.

FUTURE DIRECTIONS

These include: A scanned construction with non-coprime intersecting dimensions. Logical Pauli and Clifford structure, possibility of transversal operations. Could we design clever universal operations within a mixed-register device using some form of in-device gate teleportation scheme? Look into any possible condensed matter theory implications.

MIXED-REGISTER SCANNED CONSTRUCTION

1. Setup: Let S_1 be a stabilizer code with parameters $[[n_1, k_1, d_1]]_{Q_1}$ and S_2 be a stabilizer code with parameters $[[n_2, k_2, d_2]]_{Q_2}$ for a choice of coprime numbers Q_1, Q_2. Let $\mathcal{I}$ and $\mathcal{J}$ be finite index sets satisfying $|\mathcal{I}| = n_1$ and $|\mathcal{J}| = n_2$, and suppose $\mathcal{I} \cap \mathcal{J} \neq \emptyset$. Regard S_1 as acting on the registers indexed by $\mathcal{I}$ and S_2 as acting on the registers indexed by $\mathcal{J}$.

2. Scan: For each register $r \in \mathcal{I} \cap \mathcal{J}$, replace the local-dimension of that register by Q_1Q_2. The registers in $\mathcal{I} \setminus \mathcal{J}$ retain local-dimension Q_1, and the registers in $\mathcal{J} \setminus \mathcal{I}$ retain local-dimension Q_2.

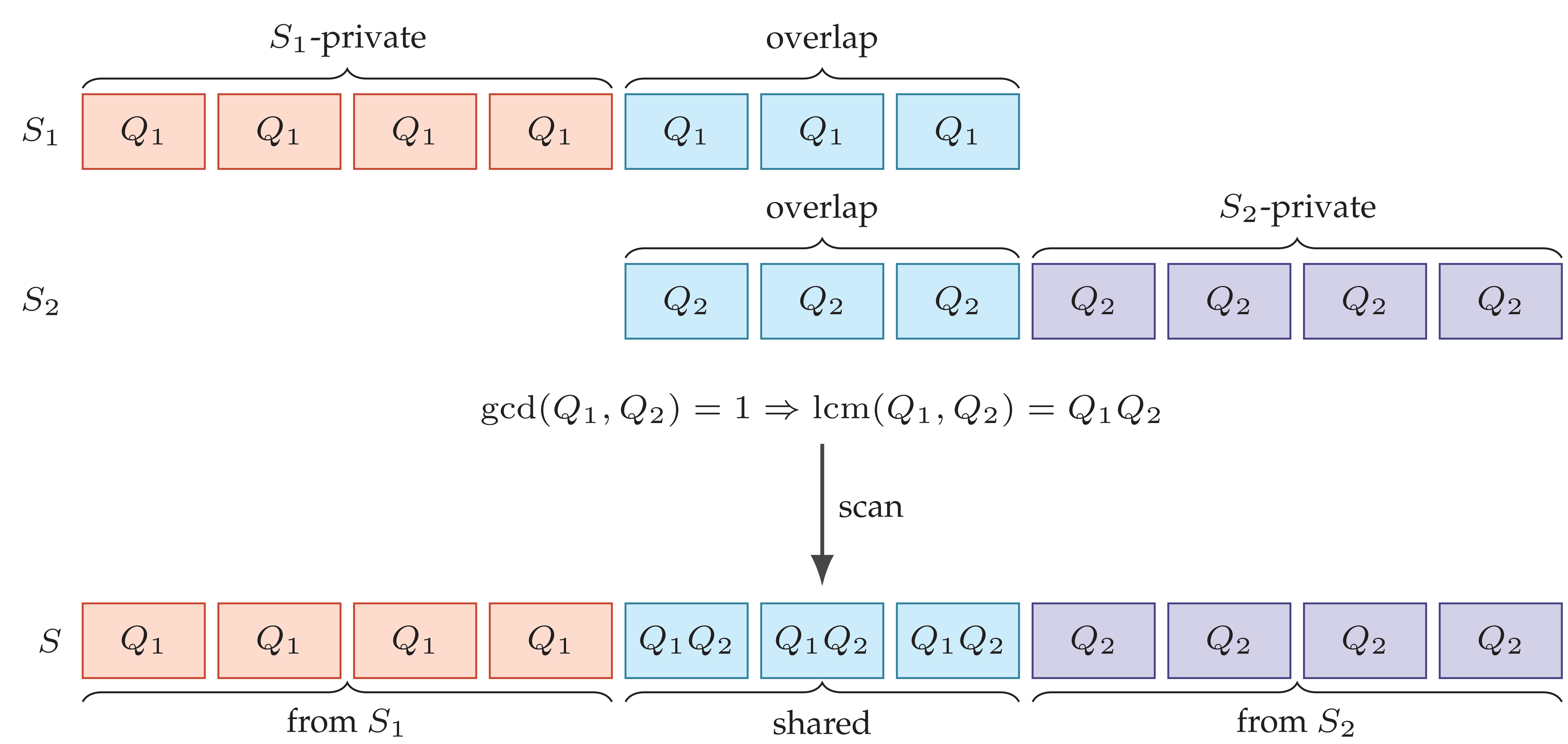
3. Result: Hence, there exists a mixed-register stabilizer code on the register set $\mathcal{I} \cup \mathcal{J}$ with local-dimensions

$$Q_1 \text{ on } \mathcal{I} \setminus \mathcal{J}, \quad Q_1Q_2 \text{ on } \mathcal{I} \cap \mathcal{J}, \quad Q_2 \text{ on } \mathcal{J} \setminus \mathcal{I}, \quad (2)$$

and parameters

$$[[n = n_1 + n_2 - |\mathcal{I} \cap \mathcal{J}|, K = Q_1^{k_1} Q_2^{k_2}, d = \min(d_1, d_2)]]_{\{Q_1, Q_1Q_2, Q_2\}}. \quad (3)$$

We achieve this by multiplying the powers of the generators on $\mathcal{I}$ in $\mathcal{I} \cap \mathcal{J}$ by Q_2, and those in $\mathcal{J}$ in $\mathcal{I} \cap \mathcal{J}$ by Q_1. This ensures commutation, while Bezout's identity ensures the distance is preserved.



- LCM registers act as arithmetic glue, coupling constituent codes into a genuinely mixed-register stabilizer code and circumventing the no-go theorem that coprime codes must be disjoint.
- Many-way gluing: Multiple pairwise coprime stabilizer codes may be simultaneously glued through shared LCM registers. Scanned constructions may themselves be scanned, generating large families of mixed-register codes.
- Optimality-preserving: Sparsity of stabilizer generators is inherited from the constituent codes. Coding-theoretically optimal constituent codes yield coding-theoretically optimal mixed-register qLDPC code families!

EXAMPLES

1. $S_1 = \langle X_2X_2 \rangle$, $S_2 = \langle X_3X_3 \rangle$. Scanning on one shared register yields

$$S = \langle X_2X_6^3I, IX_6^2X_3 \rangle, \quad (4)$$

which detects a single phase flip error. Logical operators are given by $\overline{X} = \langle IX_6I \rangle$, $\overline{Z} = \langle Z_2Z_6^{-3}I, IZ_6^{-2}Z_3 \rangle$. A member of the logical space is given by $|'0'\rangle_L = \frac{1}{\sqrt{6}}(|000\rangle + |112\rangle + |021\rangle + |130\rangle + |042\rangle + |151\rangle)$, which is not separable across the constituent register dimensions.

2. $S_1 = \langle X_2X_2 \rangle$, $S_2 = \langle X_3X_3 \rangle$, $S_3 = \langle X_4X_4 \rangle$. Suppose S_1 and S_2 share the first register, and S_2 and S_3 share the second register. Note that $\gcd(2, 4) \neq 1$. Then, by applying the scanned construction,

$$S = \langle X_2X_6^3II, IX_6^2X_{12}^4I, IIX_{12}^3X_4 \rangle, \quad (5)$$

which detects a single phase flip error. $\overline{X} = \langle IX_6II, IIX_{12}I \rangle / \langle IX_6^2X_{12}^4I \rangle$, $\overline{Z} = \langle Z_2Z_6^3II, Z_2Z_6Z_{12}^2Z_4^2, IIZ_{12}^3Z_4 \rangle$. $|'0'\rangle_L =$

$$\frac{1}{\sqrt{24}} \sum_{a=0}^{1} \sum_{b=0}^{2} \sum_{c=0}^{3} |a; 3a + 2b \bmod 6; 4b + 3c \bmod 12; c\rangle. \quad (6)$$

REFERENCES

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